Sovereign Risk and Dutch Disease*

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Abstract

I study how, in the presence of default risk, the Dutch disease amplifies an inefficiency in the sectoral allocation of capital. I develop a sovereign default model with tradable and non-tradable consumption goods, which are produced with capital, and endowments of natural resources. I show that default incentives are stronger when there is relatively more capital in the non-traded sector. Households do not internalize this when they make their portfolio choice, giving rise to over-investment in the non-traded sector. Endowments of natural resources amplify this inefficiency through the classic Dutch disease mechanism. The efficient allocation can be decentralized with a tax on returns to non-traded capital, which increases during commodity windfalls.

Keywords: Sovereign default, Dutch disease, real exchange rates

JEL Codes: F34, F41, H3, H63

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1 Introduction

There is a large literature on the effects that the Dutch disease has on economic growth, which inspired the concept of "natural resource course" coined by Auty (1993).[1] However, the relation between the Dutch disease and sovereign default risk has been less studied. This paper studies an environment in which, in the presence of default risk, the Dutch disease amplifies an inefficiency in the sectoral allocation of capital that directly affects default incentives and the borrowing terms that the government faces.

First, I develop a two-period model of a small open economy populated by a continuum of households and a benevolent government. Households have preferences for consumption of a composite aggregate of two intermediate goods: one tradable with the rest of the world and another non-tradable. Each good is produced using capital, which has to be allocated into each sector one period in advance, and before the realization of a common productivity shock. Households own all the capital in the economy and choose its sectoral allocation. The government, on the other hand, issues non-contingent debt denominated in terms of the tradable good. The debt is due in the second period and the government cannot commit to repay it. International lenders purchase the debt in the first period and consider the government's incentives to default, which depend on both the amount of debt issued and the aggregate sectoral allocation of capital.

I prove that if traded and non-traded goods are sufficiently complementary then default incentives are increasing in the share of capital allocated in the non-traded sector. The inefficiency in the model arises from the fact that households do not internalize how their sectoral allocation of capital affects the government's ability to borrow. In a competitive equilibrium, households' choices equate the expected return of capital in both sectors. However, I show that this allocation is inefficient and that households over-invest in the non-traded sector relative to the first-best alternative that a benevolent government would choose. This is because more capital in the traded sector—relative to capital in the non-traded sector—allows the government to borrow more under better terms. This result holds for any aggregate level of capital, since it is a statement on the portfolio choice. Also, the inefficiency is stronger when borrowing is more desirable, such as when the government is highly indebted and wants to roll-over debt, or during a persistent commodity

[1] Sachs and Warner (1995) document that countries with large natural resource wealth grow more slowly.

windfall, when the government wants to front-load consumption from higher future income.

I also show that the efficient allocation can be decentralized as a competitive equilibrium with an appropriate tax to capital returns in the non-traded sector that appropriately distorts households' no-arbitrage condition. This tax is proportional to the desired level of borrowing and to the sensitivity of the price of debt to the portfolio of capital between sectors.

Then, I develop a quantitative version of the two-period model with an infinite horizon, debt and capital accumulation, and commodity windfalls. In addition, households are endowed with some amount of perishable natural resources that can be sold in international markets. Commodity windfalls happen during periods when this endowment is large. Under a standard calibration, the quantitative model features the same inefficiency that was highlighted in the two-period model. Moreover, this inefficiency is stronger during commodity windfalls and a larger tax to capital returns in the non-traded sector is required to implement the efficient allocation. The optimal tax is akin to exchange rate sterilization policies where central banks accumulate foreign reserves during commodity windfalls in order to cause the real exchange rate to depreciate (or to appreciate less). This depreciation, in turn, lowers the return to capital in the non-traded sector by lowering the relative price of non-traded goods and, thus, reducing over-investment in this sector.

Finally, I present empirical evidence for the two main implications of the model: (i) "resource-rich" economies face higher default risk, which is reflected in higher interest rate spreads; and (ii) the accumulation of international reserves increases during commodity windfalls, which depreciates the real exchange rate.

Related literature.—This paper is related to the strand of literature that studies the Dutch disease and its relation to production and real exchange rates. Corden and Neary (1982) developed the benchmark model to analyze the reallocation of production factors and the process of de-industrialization. More recently, Benigno and Fornaro (2013) present a model that features episodes of abundant access to foreign capital coupled with weak productivity growth. They show that periods of large capital inflows, triggered by a fall in the interest rate, may result in inefficient outcomes in the presence of productivity externalities in the tradable sector. Alberola and Benigno (2017) study an environment in which the Dutch disease delays a commodity exporter's convergence to the world technological frontier because of the presence of an externality in dynamic productivity gains in the manufacturing sector. Ayres, Hevia, and Nicolini (2020) document that there is a strong and robust co-movement between the real exchange rates of Germany, Japan and the UK against the US dollar and a handful of primary commodities that are widely traded internationally. Benguria, Saffie, and Urzua (2021) study how commodity price super-cycles affect the economy through the (sticky) reallocation of labor. They find that models in which commodity output is an endowment only account for 45 percent of the intersectoral labor reallocation between traded and non-traded sectors. These findings suggest that the results below may be conservative because a richer model with production of commodities would induce an even bigger reallocation toward the non-traded sector, which would further amplify over-investment in it.[2]

This paper is also related to the literature that studies sovereign default risk and its relation to the production structure of the economy and commodity exports. Arellano, Bai, and Mihalache (2018) document how sovereign debt crises have disproportionately negative effects on non-traded sectors. They develop a model with capital, production in two sectors, and one period debt. The two-period model in Section 2 resembles a simplified version of their infinite horizon model. The two key differences are that I introduce exogenous endowments of commodities and, more importantly, that I analyze the inefficency that arises from different agents choosing capital allocations and debt, while in their framework all allocations are chosen by a benevolent government. Hamann, Mendoza, and Restrepo-Echavarria (2020) study the relation between oil exports, proved oil reserves, and sovereign risk. They document that sovereign risk is lower when oil production increases, but higher when reserves increase. Similarly, Esquivel (2021) documents that sovereign interest rate spreads increase substantially following news of giant oil field discoveries. Both of these papers also develop models in which a benevolent government makes all production and borrowing decisions in a centralized fashion. In a recent paper, Galli (2021) studies an environment with fiscal policy and private capital accumulation. In his environment, multiple equilibria exist where the expectations of lenders are self-fulfilling. In the bad equilibrium, pessimistic beliefs about investment make borrowing more costly. The government responds by increasing taxation, which depresses investment and makes these beliefs self-fulfilling. There are two key differences between his paper and this. First, multiplicity of equilibria is central to his analysis, while for

[2]The commodity sector would also attract production factors, which could improve the government's ability to borrow from abroad with higher income. However, the data for oil exporters suggests that sovereign spreads increase during periods of high oil prices (see Hamann, Mendoza, and Restrepo-Echavarria (2020)) and also following giant oil discoveries (see Esquivel (2021)). These papers focus on how oil production interacts with default risk, which are purposely muted in this paper to highlight the novel mechanism through over-investment in the non-traded sector.

this paper what is central is the pecuniary externality from the sectoral allocation of capital, which can be studied more clearly in an environment with a unique equilibrium. Second, he focuses on how borrowing terms depend on the absolute level of capital, while in this paper I highlight their relation to the sectoral allocation of capital. To make this point clearer, I fix the level of capital in the two-period model to turn the focus on the role of its sectoral allocation.

Layout.—Section 2 presents the two-period model and the main theoretical results; Section 3 develops the infinite-horizon model and performs the quantitative analysis; Section 4 presents some empirical evidence that supports the main implications from the models; and Section 5 concludes.

2 Two-period model

There is a small open economy with a continuum of identical households and a benevolent government. Time is discrete and there are two periods $t = 0, 1$.

Preferences and technology.—There is a final consumption good that is produced by a competitive firm with technology $Y(c_N, c_T) = \left[\omega^{\frac{1}{\eta}} c_N^{\frac{\eta-1}{\eta}} + (1-\omega)^{\frac{1}{\eta}} c_T^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$, where $\eta > 0$, $\omega \in (0,1)$ and c_T and c_N are intermediate traded and non-traded goods, respectively. This good cannot be traded with the rest of the world. The intermediate goods are produced by competitive firms using technologies $y_T = zK_T^{\alpha_T}$ and $y_N = zK_N^{\alpha_N}$; where $\alpha_T, \alpha_N \in (0,1)$, $z \in [\underline{z}, \bar{z}]$ is a common productivity shock with CDF $F(z)$, and K_T and K_N are the amounts of capital rented by firms in each sector. There is a fixed amount of capital $\bar{K}$ in the economy that is owned by the households. Capital can be allocated in the two intermediate sectors, but this allocation has to be made one period in advance. Households have preferences for consumption of the final good in each period represented by $u(c_0) + \beta\mathbb{E}[u(c_1)]$, where $\beta \in (0,1)$ is the discount factor, $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, and $\sigma > 0$. Each household owns the same amount of capital $\bar{k}$ and chooses in period 0 how much capital to allocate into each sector for period 1, subject to $k_{T,1} + k_{N,1} \leq \bar{k}$ (capital does not depreciate). The initial allocations $k_{T,0}$ and $k_{N,0}$ are given.

Commodity goods.—In each period, households are endowed with a perishable commodity good $y_{C,t} \geq 0$ that can be sold in international markets for a relative price $p_{C,t}$ (all prices are in terms of the traded intermediate good). The results of this paper hold regardless of which agent (households or government) control this endowment or whether it is costly to extract it. The results are also robust to whether $p_{C,t}$ is fixed or volatile, so for simplicity I will assume that $p_{C,t} = 1$ (this assumption will be relaxed in the quantitative analysis in Section 3). What is essential is that the available income from this commodity—like natural resources such as minerals or oil underground—is given and cannot be directly affected by the agents in the economy.[3]

Government debt and default.—There is a benevolent government that makes lump-sum transfers to the households, issues non-contingent debt B_1 in period 0 due in period 1, and lacks commitment to repay it. The debt is denominated in terms of the traded nummeraire good T and purchased at a price q by risk-neutral international lenders that behave competitively and have deep pockets. Lenders have access to a risk-free bond that pays interest r^*, which reflects their opportunity cost. At the beginning of period 1, the government observes z_1 and the capital allocations chosen by the households, $K_{T,1}$ and $K_{N,1}$, and decides whether to repay or default. If the government defaults it does not pay anything to the international lenders but productivity is penalized $z_D(z_1) \leq z_1$. I assume that z_D is increasing and continuously differentiable over $[\underline{z}, \bar{z}]$.[4]

Timing in period 0.—At the beginning of period 0 the government decides how much debt B_1 to issue. The government takes as given how this borrowing decision will affect the choices that households and firms make. The government also takes as given the price schedule q from the lenders' demand for its bonds. Once the government chooses B_1, households and firms make their individual choices taking all prices as given, as well as the aggregate allocations of capital, $K_{T,1}$ and $K_{N,1}$. Finally, lenders observe B_1, $K_{T,1}$, and $K_{N,1}$ and purchase the government bonds. This timing assumption rules out the multiplicity of equilibria studied by Galli (2021). In that environment, lenders price the government debt before investment occurs, which makes their expectations about capital in the second period self-fulfilling. Here, lenders make their pricing decisions after observing the actions of both the households and the government. [5]

[3] There is a vast literature about natural-resource extraction in which production of commodity goods is chosen by the agents through capital and/or labor allocations into these sectors (see Arezki, Ramey, and Sheng (2017), Hamann, Mendoza, and Restrepo-Echavarria (2020), and Esquivel (2021) for more references). What is common in these frameworks is an exogenous endowment of natural resources that is exploited, which is the crucial feature that is highlighted in this paper. The abstraction from the intensity of extraction is made for simplicity.

[4] Differentiability of z_D will be used in the proof of **Proposition 1** below. It is satisfied by many common assumptions for this cost, like when productivity in default is proportional $z_D = \phi z$ with $\phi \in (0,1)$. This assumption is also satisfied by $z_D = z - \max\left\{0, d_0 z + d_1 z^2\right\}$ with $d_0 < 0 < d_1$ as long as $-d_0/d_1 \leq \underline{z}$. This quadratic formulation was introduced by Chatterjee and Eyigungor (2012) in a pure exchange economy and is widely used in the litterature.

[5] Cole and Kehoe (2000) study the case in which lenders price the bonds before observing government borrowing, which is another source of multiplicity that I rule out with this timing assumption.

Firms.—All firms behave competitively and maximize profits. From the maximization problem of the final good producer we get the demands for intermediate goods are:

$$c_{N,t} = \omega \left(\frac{P_t}{p_{N,t}} \right)^{\eta} Y_t \tag{1}$$

$$c_{T,t} = (1-\omega)(P_t)^{\eta} Y_t \tag{2}$$

where $p_{N,t}$ is the relative price of the non-traded intermediate. Since the production function features constant-returns to scale, I assume that the firm makes zero profits and $P_t = \left[\omega p_{N,t}^{1-\eta} + (1-\omega)\right]^{\frac{1}{1-\eta}}$ is the price index reflecting the cost of the final consumption good. From the maximization problem of the intermediate good producers we get that the rental rates of capital in each sector are:

$$r_{N,t} = p_{N,t} \alpha_N z_t (K_{N,t})^{\alpha_N - 1} \tag{3}$$

$$r_{T,t} = \alpha_T z_t (K_{T,t})^{\alpha_T - 1} \tag{4}$$

for a given capital allocation and productivity shock. Profits from intermediate-goods firms are rebated back to the households.

Households.—Each household owns the same amount of capital $\bar{k}$. Since all households are identical, they do not trade capital with each other and I assume capital cannot be sold to foreigners. The problem of a representative household is:

$$\max_{k_{N,1}, k_{T,1}} u(c_0) + \beta \mathbb{E}[u(c_1)] \tag{5}$$

$$\begin{aligned} s.t. \quad P_0 c_0 &= r_{N,0} k_{N,0} + r_{T,0} k_{T,0} + y_{C,0} + \Pi_0 + G_0 \\ P_1 c_1 &= r_{N,1} k_{N,1} + r_{T,0} k_{T,1} + y_{C,1} + \Pi_1 + G_1 \\ k_{N,1} + k_{T,1} &\leq \bar{k} \end{aligned}$$

where Π_t are the profits of the intermediate goods firms and G_t is a government transfer. In period 0, the transfer is $G_0 = q_0 B_1 - B_0$. If the government defaults in period 1 then $G_1 = 0$, and if the government repays $G_1 = -B_1$.[6] The expectation integrates over $F(z)$ and the households have some

[6] Note that, if the government owned the natural resources then $y_{C,0}$ and $y_{C,1}$ would be on the government's budget constraint, instead of the households. However, since it is a given endowment, the government would just include

beliefs about the aggregate capital allocation and the government's default choice. Subsection 2.2 reformulates this problem in recursive form and discusses these beliefs in detail.

2.1 Static allocations

The dynamic choices in the model are borrowing and the sectoral allocation of capital. These are the key to analyze the inherent inefficiency and its relation to the endowment of commodities. This subsection uses optimality conditions from the firms and market clearing conditions for goods to characterize all other allocations and prices as functions of the dynamic choices, which will extremely simplify notation.

Let $\lambda_t \in (0,1)$ be the share of $\bar{k}$ that a representative household allocates to the traded sector T for period t, and Λ_t the share of aggregate capital $\bar{K}$ allocated in T. The aggregate state of the economy in a given period is (z_t, x_t), where $x_t = (\Lambda_t, B_t)$. I will treat z_0, λ_0, Λ_0, B_0, $y_{C,0}$, and $y_{C,1}$ as fixed parameters. For simplicity, I assume that the legacy debt B_0 is low enough so that the government would not want to default on it at the beginning of period $t = 0$.

Consumption in period 0.—Since λ_0 and Λ_0 are given, aggregate consumption in period 0 is given by:

$$C_0(x_1) = Y\left(y_N\left(z_0, (1-\Lambda_0)\bar{K}\right), y_T\left(z_0, \Lambda_0 \bar{K}\right) + y_{C,0} + qB_1 - B_0\right) \tag{6}$$

where q is the price of newly issued debt. As discussed in the next subsection, q is a function of Λ_1 and B_1. Note that consumption in $t = 0$ can only change through borrowing. Since borrowing terms only depend on aggregate variables, households behave taking $c_0 = C_0$ as given.

Consumption in period 1.—Given an aggregate state (z, x), market clearing implies that, if the government decides to repay, consumption of the traded good is $c_T^P(z,x) = z\left(\Lambda\bar{K}\right)^{\alpha_T} + y_{C,1} - B$ and consumption of the non-traded good is $c_N^P(z,x) = z\left((1-\Lambda)\bar{K}\right)^{\alpha_N}$. Similarly, if the government defaults these quantities are $c_T^D(z,x) = z_D(z)\left(\Lambda\bar{K}\right)^{\alpha_T} + y_{C,1}$ and $c_N^D(z,x) = z_D(z)\left((1-\Lambda)\bar{K}\right)^{\alpha_N}$, respectively. It is clear that in default consumption of the non-traded good drops because of the productivity penalty, but consumption of the traded good may increase because there is no debt to repay. Given these equations, aggregate consumption in repayment is $C^P(z,x) = Y\left(c_N^P(z,x), c_T^P(z,x)\right)$ and in default $C^D(z,x) = Y\left(c_N^D(z,x), c_T^D(z,x)\right)$.

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them in the lump-sum transfers, which would yield an isomorphic problem.

Prices in period 1.—From equations 1 and 2 we get that the price of the non-traded intermediate in repayment is $p_N^P(z,x) = \left(\frac{\omega}{1-\omega}\frac{c_T^P}{c_N^P}\right)^{\frac{1}{\eta}}$ and in default it is $p_N^D(z,x) = \left(\frac{\omega}{1-\omega}\frac{c_T^D}{c_N^D}\right)^{\frac{1}{\eta}}$. These imply that the price index of the final consumption good is $P^P(z,x) = \left[\omega\left(p_N^P\right)^{1-\eta} + (1-\omega)\right]^{\frac{1}{1-\eta}}$ in repayment and $P^D(z,x) = \left[\omega\left(p_N^D\right)^{1-\eta} + (1-\omega)\right]^{\frac{1}{1-\eta}}$ in default. Finally, from the optimality conditions for the intermediate firms' problems we get that the rental rates of capital in repayment are $r_T^P(z,x) = \alpha_T z(\Lambda\bar{K})^{\alpha_T-1}$ and $r_N^P(z,x) = p_N^P\alpha_N z((1-\Lambda)\bar{K})^{\alpha_N-1}$ in the traded and non-traded sectors, respectively. Similarly, these rental rates in default are $r_T^D(z,x) = \alpha_T z_D(z)(\Lambda\bar{K})^{\alpha_T-1}$ and $r_N^D(z,x) = p_N^D\alpha_N z_D(z)((1-\Lambda)\bar{K})^{\alpha_N-1}$.

2.2 Decentralized equilibrium

Given the optimal static allocations defined in the previous subsection, the default decision and dynamic problems of the government and of a representative household are defined as follows.

Default decision.—At the beginning of period 1, the government observes (z,x) and solves the default problem:

$$\max\left\{V^P(z,x), V^D(z,x)\right\} \tag{7}$$

where the value of repayment is $V^P(z,x) = u\left(C^P(z,x)\right)$ and the value of default is $V^D(z,x) = u\left(C^D(z,x)\right)$. Given x, the default set is characterized by a cutoff value $z^*(x)$ such that:

$$V^P(z^*(x),x) = V^D(z^*(x),x) \tag{8}$$

Household's problem.—In period 0, households make their decisions after the government has issued B. As mentioned before, households behave as if their actions do not affect consumption in $t=0$, since it can only change with the value of borrowed resources qB, which households take as given. Given this, the problem of a representative household is:

$$
\begin{aligned}
\max_{\lambda} \int_{\underline{z}}^{z^*(x)} \beta u\left(c^D\right) dF(z) + \int_{z^*(x)}^{\bar{z}} \beta u\left(c^P\right) dF(z) \qquad (9)\\
s.t. \quad P^D c^D = \left[r_N^D (1-\lambda) + r_T^D \lambda\right] \bar{k} + y_{C,1} + \Pi^D \\
P^P c^P = \left[r_N^P (1-\lambda) + r_T^P \lambda\right] \bar{k} + y_{C,1} + \Pi^P - B \\
\Lambda = \Gamma_H(B)
\end{aligned}
$$

where $x = (\Lambda, B)$, Π^D and Π^P are profits made in $t = 1$ by all firms in default and repayment, respectively, all prices depend on (z, x) as defined in Subsection 2.1, and Γ_H is the household's belief about the aggregate capital allocation for $t = 1$. Denote the policy function of a representative household as $\lambda^*(B)$.

Debt issuance.—At the beginning of period 0, the government chooses debt issuance B to solve:

$$
\begin{aligned}
\max_{B} u(C_0(x)) + \beta \int_{\underline{z}}^{z^*(x)} u\left(C^D(z,x)\right) dF(z) + \beta \int_{z^*(x)}^{\bar{z}} u\left(C^P(z,x)\right) dF(z) \qquad (10)\\
s.t. \quad \Lambda = \lambda^*(B)
\end{aligned}
$$

where C_0, C^D and C^P depend on (z, x) as defined in Subsection 2.1, and Γ_G is the government's belief about the aggregate capital allocation for $t = 1$. Unlike the representative household, the government does internalize how its choice of B affects the capital allocation for next period through Γ_G. Denote the solution to the maximization problem as B^*.

DEFINITION 1: (Decentralized Equilibrium) A decentralized equilibrium is a policy function for the households $\lambda^*(B)$, a debt issuance for the government B^*, a price schedule $q(x)$, and beliefs $\Gamma_H(B)$ and $\Gamma_G(B)$ such that: (i) given q, Γ_H, and Γ_G, the policy function λ^* solves the household's problem for any B, and the allocation B^* solve the government's problem; (ii) the beliefs are consistent $\Gamma_G(B) = \Gamma_H(B) = \lambda^*(B)$, and (iii) the price schedule q satisfies the lenders' no-arbitrage condition:

$$
q(x) = \frac{1 - F(z^*(x))}{1 + r^*} \qquad (11)
$$

DEFINITION 2: (Equilibrium Allocation) An equilibrium allocation is $\tilde{x} = \left(\tilde{\Lambda}, \tilde{B}\right)$ such that

$\tilde{B} = B^*$ and $\tilde{\Lambda} = \lambda^*(B^*)$.

2.3 Efficiency

Given a state (z,x) in period 1, a benevolent social planner would face the same default problem as the government in 7 (and, hence, the same price schedule $q(x)$ in period 0). However, in period 0 the planner simultaneously chooses Λ and B to solve:

$$\max_x u(C_0(x)) + \beta \int_{\underline{z}}^{z^*(x)} u\left(C^D(z,x)\right) dF(z) + \beta \int_{z^*(x)}^{\bar{z}} u\left(C^P(z,x)\right) dF(z) \tag{12}$$

where the key difference is that the planner chooses Λ directly, as opposed to the government who can only indirectly affect it through its choice of B.

Definition 3: (Efficient Allocation) An allocation $\hat{x} = \left(\hat{\Lambda}, \hat{B}\right)$ is efficient if it solves the social planner's problem in 12.

2.4 Decentralization

As can be seen in equations (8) and (11), the borrowing terms depend on the amount of debt issued B and on the capital allocation between the two intermediate sectors, summarized by Λ.[7] Given a level of debt issuance $\tilde{B}$, the capital allocation in the decentralized equilibrium $\tilde{\Lambda}$ is pinned down by the Euler equation of a representative household:

$$\mathbb{E}\left[\beta u'\left(\tilde{C}_1\right) \frac{(\tilde{r}_T - \tilde{r}_N)\bar{K}}{\tilde{P}}\right] = 0 \tag{13}$$

where, to ease notation, "tildes" indicate prices and allocations consistent with the decentralized equilibrium.[8] Equation 13 is the no-arbitrage condition in which the expected returns to allocating capital in either sector are equated. On the other hand, the Euler equation for Λ from the social

[7] These terms would also depend on the level of capital installed for $t = 1$, as studied by Gordon and Guerron-Quintana (2018), Arellano, Bai, and Mihalache (2018), Galli (2021), and Esquivel (2021). The assumption that the total amount of capital is fixed allows me to highlight the role of the sectoral allocation. This assumption will be relaxed in the quantitative analysis.

[8] Formally, this expression is $\int_{\underline{z}}^{z^*(\tilde{x})} \beta u'\left(C^D(z,\tilde{x})\right) \frac{r_T^D(z,\tilde{x}) - r_N^D(z,\tilde{x})}{P^D(z,\tilde{x})} dF(z) + \int_{z^*(\tilde{x})}^{\bar{z}} \beta u\left(C^P(z,\tilde{x})\right) \frac{r_T^P(z,\tilde{x}) - r_N^P(z,\tilde{x})}{P^P(z,\tilde{x})} dF(z) = 0$. See Appendix A for the formal derivation of this equation.

planner's problem can be written as:

$$\mathbb{E}\left[\beta u'\left(\hat{C}_1\right)\frac{\left(\hat{r}_T-\hat{r}_N\right)\bar{K}}{\hat{P}}\right]+u'\left(\hat{C}_0\right)\frac{\partial \hat{q}}{\partial \Lambda}\frac{\hat{B}}{\hat{P}_0}=0 \tag{14}$$

where I use "hats" for variables that correspond to the efficient allocation. Here $\hat{r}_T$ and $\hat{r}_N$ are the marginal products of capital in each sector, and $1/\hat{P}$ and $1/\hat{P}_0$ are the marginal products of c_T in the production of the final good in each period.[9] Like in equation 13, these marginal products are evaluated in default or repayment, depending on the sate.

The second term in equation 14 shows that the planner also takes into account how Λ affects the borrowing terms and consumption in period 0. The sign of this second term depends entirely on whether Λ improves or worsens borrowing terms. Proposition 1 shows how default incentives respond to the aggregate capital allocation, which is the main result of this section.

PROPOSITION 1: If $\eta < 1$, then the default set is shrinking in Λ. That is, $\frac{\partial z^*(x)}{\partial \Lambda} \leq 0$.

Proof: First, note that if the state x is such that $V^P > V^D$ or $V^P < V^D$ for all $z \in [\underline{z}, \bar{z}]$ then $\frac{\partial z^*(x)}{\partial \Lambda} = 0$. For any x such that $z^*(x) \in (\underline{z}, \bar{z})$ we can fully differentiate equation (8) with respect to Λ and rearranging we get

$$\frac{\partial z^*(x)}{\partial \Lambda} = -\frac{\frac{\partial V^P(z^*(x),x)}{\partial \Lambda} - \frac{\partial V^D(z^*(x),x)}{\partial \Lambda}}{\frac{\partial V^P(z^*(x),x)}{\partial z} - \frac{\partial V^D(z^*(x),x)}{\partial z}} \tag{15}$$

where the numerator is the difference of marginal values of Λ in repayment and default and the denominator is the difference between the marginal value of productivity in repayment and default. The proof proceeds in two steps: the first shows that the denominator is positive and the second that the numerator is also positive. Given that both numerator and denominator are positive, the result follows.

Step 1: For any $\varepsilon > 0$ we have $V^P(z^*(x)+\varepsilon, x) > V^D(z^*(x)+\varepsilon, x)$ (this follows from continuity and monotonicity of u, all production functions, and of z_D). Then $\frac{V^P(z^*(x)+\varepsilon,x)-V^P(z^*(x),x)}{\varepsilon} > \frac{V^D(z^*(x)+\varepsilon,x)-V^D(z^*(x),x)}{\varepsilon}$ and taking the limit as $\varepsilon \to 0$ we get $\frac{\partial V^P(z^*(x),x)}{\partial z} > \frac{\partial V^D(z^*(x),x)}{\partial z}$. The argument for $\varepsilon < 0$ follows identically.

Step 2: Recall that $V^P(z,x) = u\left(Y\left(c_N^P(z,x), c_T^P(z,x)\right)\right)$ and $V^D(z,x) = u\left(Y\left(c_N^D(z,x), c_T^D(z,x)\right)\right)$

[9] Since all firms are competitive, these marginal products have the same form as the prices in the decentralized economy, which allows for this compact and intuitive notation. Formally, the only price in the social planner's problem is q (see Appendix A for the formal derivation of this equation).

where consumption quantities of intermediates are as defined above in Subsection 2.1. Then, we can write the two terms in the numerator of equation (15) as:

$$\frac{\partial V^P}{\partial \Lambda} = u'\left(Y^P\right)\left[\frac{\partial Y^P}{\partial c_T}\frac{y_T^P}{\Lambda}\alpha_T - \frac{\partial Y^P}{\partial c_N}\frac{y_N^P}{1-\Lambda}\alpha_N\right] \tag{16}$$

$$\frac{\partial V^P}{\partial \Lambda} = u'\left(Y^D\right)\left[\frac{\partial Y^D}{\partial c_T}\frac{y_T^D}{\Lambda}\alpha_T - \frac{\partial Y^D}{\partial c_N}\frac{y_N^D}{1-\Lambda}\alpha_N\right] \tag{17}$$

where superscripts P and D in the right-hand-side indicate that the functions (or their derivatives) are evaluated at the repayment and default state, respectively. Intuitively, the marginal value of Λ (either in default or in repayment) is the marginal product of the additional unit of capital in the traded sector minus the marginal product of the unit of capital that is withdrawn from the non-traded sector. Now, note that at $z^*(x)$ we have $Y^P = Y^D = Y^*$ by definition. Subtracting equations (16) and (17) and rearranging terms we get:

$$\begin{aligned}
\frac{\partial V^P}{\partial \Lambda} - \frac{\partial V^D}{\partial \Lambda} &= u'(Y^*)\left\{\left[\frac{\partial Y^P}{\partial c_T}y_T^P - \frac{\partial Y^D}{\partial c_T}y_T^D\right]\frac{\alpha_T}{\Lambda} + \left[\frac{\partial Y^D}{\partial c_N}y_N^D - \frac{\partial Y^P}{\partial c_N}y_N^P\right]\frac{\alpha_N}{1-\Lambda}\right\} \\
&= u'(Y^*)\left\{\left[\underbrace{\left(\frac{1}{c_T^P}\right)^{\frac{1}{\eta}} z^* - \left(\frac{1}{c_T^D}\right)^{\frac{1}{\eta}} z_D^*}_{>0}\right]\frac{y_T^P}{z^*}\frac{(1-\omega)^{\frac{1}{\eta}}\alpha_T}{\Lambda} + \left[\underbrace{\left(\frac{1}{c_N^D}\right)^{\frac{1-\eta}{\eta}} - \left(\frac{1}{c_N^P}\right)^{\frac{1-\eta}{\eta}}}_{>0 \text{ if } \eta<1}\right]\frac{(\omega)^{\frac{1}{\eta}}\alpha_N}{1-\Lambda}\right\}(Y^*)^{\frac{1}{\eta}}
\end{aligned} \tag{18}$$

where the crucial step for the second equality is to note that consumption of non-traded goods equals production but consumption of traded goods does not. Note that since $z_D^* \leq z^*$ then $c_N^D \leq c_N^P$, so for $Y^D = Y^P$ it must be the case that $c_T^D \geq c_T^P$. Given this, we get that the first term in the bracket of equation (18) is positive. The second term is also positive as long as $\eta < 1$.□

Intuitively, there are two effects that Λ has on default incentives. The first can be though of as the classic *income effect*, where having more capital installed in the traded sector makes servicing the debt less painful in states where productivity is low (recall that total capital is fixed, so a large Λ does not necessarily improve the value of autarky). The second, a *substitution effect*, is more subtle and has to do with how the default decision affects the mix of intermediate goods that is consumed. Upon default, the productivity cost reduces c_N and reneging on the debt increases c_T. If intermediate goods are complements (low η), the utility value of this change in the bundle is low and, importantly, lower with high Λ because c_N is already low and its marginal value high. When they are "substitutes enough" (large enough η) then the increase in c_T by defaulting could overcome the lower c_N from the productivity cost.

From Proposition 1 it follows that borrowing terms improve with a larger share of capital allocated to the traded sector:

$$\frac{\partial q}{\partial \Lambda} = -\frac{f\left(z^*\left(x\right)\right)}{1+r^*}\frac{\partial z^*\left(x\right)}{\partial \Lambda} \geq 0$$

where f is the PDF of z. The sufficient condition for this to hold is that traded and non-traded intermediates are "sufficiently complements", that is, that the elasticity of substitution η is less than 1.[10]

Proposition 2: (*Misallocation*) For any given level of debt issuance B_1, households in the decentralized equilibrium overinvest in the non-traded sector and underinvest in the traded one.

Proof: See Appendix B.

From Proposition 1 it follows that the second term in equation 14 is positive for any $\hat{B} \geq 0$, so with the planner's optimal allocation we get:

$$\mathbb{E}\left[\beta u'\left(\hat{C}_1\right)\frac{\left(\hat{r}_T - \hat{r}_N\right)\bar{K}}{\hat{P}_1}\right] \leq 0$$

where we can interpret the second term in equation 14 as a "wedge" that represents the degree of disagreement for the capital allocation between the social planner and the private sector. Intuitively, the planner foregoes higher expected returns in the non-traded sector in period 1 in order to have a higher ability to borrow in period 0 (q is increasing in Λ). Households do not internalize this effect on the borrowing terms, so they choose to continue to allocate capital in the non-traded sector until the expected returns are equated, as indicated by equation 13. An immediate corollary of Proposition 1 is that the equilibrium allocation is not efficient.

Proposition 3: (*Optimal tax*) The government can implement the efficient allocation as a decentralized equilibrium by imposing the following tax on returns to investment in the non-traded sector:

$$\tau^* = \frac{u'\left(\hat{C}_0\right)\frac{\widehat{\partial q}}{\partial \Lambda}\frac{\hat{B}_1}{\hat{P}_0}}{\mathbb{E}\left[\beta u'\left(\hat{C}_1\right)\frac{\hat{r}_{N,1}\bar{K}}{\hat{P}_1}\right]} \tag{19}$$

[10]The range of estimates for this elasticity is between 0.4 and 0.83, so this sufficient condition would hold in any standard parametrization (see Stockman and Tesar (1995), Mendoza (2005), and Bianchi (2011)).

Proof: Obvious after multiplying $\tilde{r}_{N,1}$ by $(1-\tau_N^*)$ in equation 13.

If we define the real exchange rate as the price of the traded good in terms of the domestic final consumption good $\xi = \frac{1}{P}$, then the optimal tax is inversely proportional to the expected real depreciation $\frac{\xi_1}{\xi_0}$ and proportional to the desired borrowing level $\hat{B}_1$. Also note that, absent default risk $\tau^* = 0$ since $q = \frac{1}{1+r^*}$ would be constant and $\frac{\partial q}{\partial \Lambda} = 0$. This implies that the inefficiency presented in this section only arises in the presence of default risk.

3 Quantitative analysis

The infinite horizon version of the model falls into the class of models that follow Eaton and Gersovitz (1981). I continue to make assumptions to rule out common sources of multiplicity to keep the analysis focused on the role of sectoral capital allocations. However, there may be a rich set of multiplicity of equilibria that could be worth exploring in future research (see Galli (2021) for a discussion of multiplicity in default models with capital, and Cole and Kehoe (2000) and Aguiar, Chatterjee, Cole, and Stangebye (2022) for discussions of multiplicity in models with long-term debt).

3.1 Environment

This is an extension of the model from Section 2 to an infinite horizon economy with long-term debt, capital accumulation, and commodity windfalls. Any missing details are the same as in the two-period version.

Preferences and technology.—Production technologies for the intermediate and final goods remain the same. Preferences for the final good are now $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$. There are two stocks of capital, one for the non-traded sector K_N and one for the traded sector K_T, each stock depreciates at a rate δ. The final good is used for production and investment. The investment goods are produced by competitive firms who pay capital adjustment costs $\Psi(K_{i,t+1}, K_{i,t})$ with $i \in \{N, T\}$. Denote the prices of investment goods as $q^k_{j,t} = 1 + \Psi_1(K_{i,t+1}, K_{i,t})$. Productivity follows an AR(1) process $z_t = \rho \log z_{t-1} + \varepsilon_t$ with $\varepsilon_t \sim N(0, \sigma_z^2)$.

Commodity windfalls.—Households receive each period an endowment of the perishable commodity good $y_{C,t}$ which can take one of two values y_L or y_H with $y_L < y_H$. The commodity endowment follows a Markov chain with transition probabilities $\pi_{h,j}$, $h, j \in \{L,H\}$.

Households.—Households own all firms and capital in the economy. In period t, the budget constraint of a household is

$$c_t + \sum_{h=N,T} q^k_{h,t} i_{h,t} \leq \sum_{h=N,T} \left(r_{h,t} k_{h,t} + (1-\delta) q^k_{h,t} k_{h,t} \right) + y_{C,t} + \Pi_t + G_t \tag{20}$$

where all prices, profits, and the government transfer G_t depend on aggregate allocations and whether the government is in good or bad financial standing and are defined in the same way as in Subsection 2.1. Each period, households make individual consumption and investment decisions to maximize their utility subject to their budget constraint. Households take all prices as given, as well as all present and future government policies.

Government debt and default.—The government issues long-term debt and lacks commitment to repay it. Debt matures at a rate γ and pays a coupon κ on the debt that has not yet matured. The government budget constraint is:

$$G_t + (\gamma + \kappa (1-\gamma)) B_t = q_t \left(B_{t+1} - (1-\gamma) B_t \right)$$

where q_t is the market price of the government debt. As in the two-period model, the government takes household's behavior and the price schedule q as given. If the government defaults then productivity is $z_D(z) = z - \max\left\{0, d_0 z + d_1 z^2\right\}$, with $d_0 < 0 < d_1$, and the government gets excluded from financial markets for a random number of periods. The government gets readmitted with probability θ and all debt that was defaulted on is forgiven. During exclusion $G_t = 0$.

Timing within a period.—At the beginning of a period all shocks are realized. Then, the government observes the shocks, the aggregate capital stocks $K_{N,t}$ and $K_{T,t}$, and the current stock of debt B_t and decides whether to default or not. If the government does not default then it decides how much debt to issue B_{t+1}. Then, households observe B_{t+1} and all shocks and aggregate capital allocations and make their investment and consumption decisions. Finally, lenders observe B_{t+1} and aggregate investment and price the government debt.

3.2 Recursive formulation and equilibrium

The aggregate state of the economy is now (s,x), where $s=(z,y_C)$, $x=(K,B)$, and $K=(K_N,K_T)$. The individual state of a household is $k=(k_N,k_T)$.

Households.—The value of a representative household in repayment is:

$$H^P(s,k,x;G)=\max_{c,i_N,i_T,k'}\left\{u(c)+\beta\mathbb{E}\left[H^D\left(s',k',K'\right)|d'=1\right]+\beta\mathbb{E}\left[H^P\left(s',k',x';G'\right)|d'=0\right]\right\}$$

subject to the household's budget constraint 20 in repayment and the laws of motion of capital $k'_j=i_j+(1-\delta)k_j$. The expectations are conditional on the government's default decision in the following period d', which depends on the aggregate state then.[11] The representative household understands $G=q(s,x')(B'-(1-\gamma)B)-(\gamma+\kappa(1-\gamma))B$ and has beliefs for the law of motion of the aggregate state denoted by $x'=\Gamma_H^P(s,x;G)$ and beliefs about future government transfers $G'=\Gamma_H^G(s',x')$. Denote the household's policy function as $\tilde{k}^P(s,k,x;G)$.

The value of the household when the government is in the default state is:

$$H^D(s,k,K)=\max_{c,i_N,i_T,k'}\left\{u(c)+\beta\theta\mathbb{E}\left[H^P\left(s',k',\left(K',0\right);G'\right)\right]+\beta(1-\theta)\mathbb{E}\left[H^D\left(s',k',K'\right)\right]\right\}$$

subject to the household's budget constraint in default and the laws of motion of capital. To ease notation, I (correctly) assume that, upon reentry into financial markets, the government will not default on 0 debt. The household's beliefs for the law of motion of the aggregate state in this case are $x'=\Gamma_H^D(s,K)$. Denote the policy function as $\tilde{k}^D(s,k,K)$.

Government.—At the beginning of a period in good standing the value of the government is:

$$G(s,x)=\max_{d\in\{0,1\}}\left\{dG^D(s,K)+(1-d)G^P(s,x)\right\}$$

where G^D is the value of defaulting and G^P is the value of repayment. Since the government is

[11] Formally, the continuation value is $\beta\sum_{j\in H,L}\pi^{ij}\left[\int_{\underline{z}}^{z^*\left(y_C^j,x'\right)}H^D(s',k',K')\,dF(z'|z)+\int_{z^*\left(y_C^j,x'\right)}^{\bar{z}}H^P(s',k',x';G')\,dF(z'|z)\right]$, where z^* denotes the cut-off productivity level under which the government would default given a future realization for y_C and the aggregate state x'.

benevolent, the value of defaulting is the value of a representative household:

$$G^{D}(s,K)=H^{D}(s,K,K)$$

where $k=K$ since all households are identical. If the government decides to repay, then the value is:

$$\begin{aligned}
&G^{P}(s,x)=\max_{B',G}H^{P}(s,K,x;G)\\
s.t.\qquad &G=q\left(s,x'\right)\left(B'-(1-\gamma)B\right)-(\gamma+\kappa(1-\gamma))B\\
&K'=\Omega(s,x;G)
\end{aligned}$$

where Ω is the government's belief about how the aggregate stock of capital is going to evolve given its choice for G and B'. In equilibrium, this will coincide with the representative household's policy function for capital. Denote the policy functions of the government as $B(s,x)$ and $G(s,x)$.

DEFINITION 4: (Recursive Competitive Equilibrium) A recursive competitive equilibrium is policy functions for the households $\tilde{k}^{P}(s,k,x;G)$ and $\tilde{k}^{D}(s,k,K)$, policy functions for the government $B(s,x)$ and $G(s,x)$, a price schedule $q(s,x')$, beliefs for the households $\Gamma_{H}^{P}(s,x;G)$, $\Gamma_{H}^{G}(s,x)$ and $\Gamma_{H}^{D}(s,K)$, and beliefs for the government $\Omega(s,x;G)$ such that: (i) given q, and the household's and government's beliefs, the policy functions solve the household's problem for any G and the government's problem; (ii) the beliefs are consistent:

$$\begin{aligned}
\Gamma_{H}^{P}(s,x;G)&=\left(\tilde{k}^{P}(s,K,x;G),B(s,x)\right)\\
\Gamma_{H}^{G}(s,x)&=G(s,x)\\
\Gamma_{H}^{D}(s,K)&=\tilde{k}^{D}(s,K,K)\\
\Omega(s,x;G)&=\tilde{k}^{P}(s,K,x;G)
\end{aligned}$$

, and (iii) the price schedule q satisfies the lenders' no-arbitrage condition:

$$q\left(s,x'\right)=\frac{\mathbb{E}_{s'|s}\left[(1-d')(\gamma+(1-\gamma)(\kappa+q(s',x'')))\right]}{1+r^{*}}$$

where d' and x'' are given by the government's policy functions and the laws of motion of aggregate capital.

3.3 Efficiency

As in the two-period model, I analyze the problem of a benevolent social planner to define efficiency. The value of the central planner in good financial standing is:

$$V(s,x) = \max_{d} \left\{ dV^D(s,K) + (1-d)V^P(s,x) \right\}$$

where d is the default decision and V^D and V^P are the value of defaulting and repaying, respectively. The value of defaulting is

$$\begin{aligned}
V^D(s,K) &= \max_{K'} \left\{ u\left(C^D(s,K)\right) + \beta\theta\mathbb{E}\left[V\left(s',\left(K',0\right)\right)\right] + \beta(1-\theta)\mathbb{E}\left[V^D\left(s',K'\right)\right] \right\} \\
s.t. \quad c + \sum_{j=N,T} i_j &\leq Y^D(s,x,y_C) - \sum_{j=N,T} \Psi\left(K'_j, K_j\right) \\
K'_j &= i_j + (1-\delta)K_j
\end{aligned}$$

where Y^D is aggregate production defined in Subsection 2.1. Denote the policy function as $\hat{K}^D(s,K)$. The value of repayment is:

$$\begin{aligned}
V^P(s,x) &= \max_{c,x'} \left\{ u(c) + \beta\mathbb{E}\left[V\left(s',x'\right)\right] \right\} \\
s.t. \quad c + \sum_{j=N,T} i_j &\leq Y(s,x,T) - \sum_{j=N,T} \Psi\left(K'_j, K_j\right) \\
K'_j &= i_j + (1-\delta)K_j \\
T &= y_C + \hat{q}\left(s,x'\right)\left[B' - (1-\gamma)B\right] - (\gamma + \kappa(1-\gamma))B
\end{aligned}$$

where $\hat{q}$ is the price schedule for the planner's debt. Denote the policy function as $\hat{x}(s,x)$. Lender's price the planner's debt according to

$$\hat{q}\left(s,x'\right) = \frac{\mathbb{E}_{s'|s}\left[\left(1-\hat{d}'\right)(\gamma + (1-\gamma)(\kappa + \hat{q}(s',\hat{x}'')))\right]}{1+r^*}$$

where $\hat{d}'$ and $\hat{x}''$ are the planner's policies.

DEFINITION 5: (Efficient Allocations) Given a state (s,x), an allocation $\left(\hat{K}'_N, \hat{K}'_T, \hat{B}'\right)$ is efficient if it coincides with the planners policy functions.

From the household's Euler equations in repayment we can derive a no-arbitrage condition for both types of capital:

$$0 = \mathbb{E}_t \left[\frac{\beta u'\left(\tilde{c}_{t+1}\right)}{u'\left(\tilde{c}_t\right)} \left(\tilde{R}_{T,t+1} - \tilde{R}_{N,t+1}\right) \right] \tag{21}$$

where $\tilde{R}_T$ and $\tilde{R}_N$ are the net returns to capital for the tradable and non-tradable sectors, respectively, and $\tilde{c}$ are the households consumption choices. The expectation is conditional on information at period t and considers the government's default policy in $t+1$. Similarly, assuming that $\hat{q}$ is differentiable, we get from the Euler equations from the planner's problem in repayment:

$$0 = \mathbb{E}_t \left[\frac{\beta u'\left(\hat{C}_{t+1}\right)}{u'\left(\hat{C}_t\right)} \left(\hat{R}_{T,t+1} - \hat{R}_{N,t+1}\right) \right] + \left[\frac{\partial \hat{q}}{\partial K_T} - \frac{\partial \hat{q}}{\partial K_N} \right] \frac{\hat{B}_{t+1} - (1-\gamma) B_t}{\hat{P}_t} \tag{22}$$

where $\hat{P}_t$ is the price index of the final good, $\hat{R}_T$ and $\hat{R}_N$ are the net returns to capital in each sector, $\hat{C}$ is the planner's consumption decision, and $\frac{\partial \hat{q}}{\partial K_T}$ and $\frac{\partial \hat{q}}{\partial K_N}$ are the derivatives of the price of the debt with respect to each type of capital.

The difference between $\frac{\partial \hat{q}}{\partial K_T}$ and $\frac{\partial \hat{q}}{\partial K_N}$ is akin to $\frac{\partial q}{\partial \Lambda}$ in the two-period model, where capital was fixed; indicating the total change in the price of the debt from shifting capital from the non-traded to the traded sector (all else constant). The second term in equation 22 will be positive if debt issuance is positive and if the price of the debt increases more with capital in the traded sector than with capital in the non-traded sector. From these equations we can compute the optimal tax (or subsidy, depending on the sign of $\frac{\partial \hat{q}}{\partial K_T} - \frac{\partial \hat{q}}{\partial K_N}$) to returns to capital in the non-traded sector:

$$\tau_t^* = \frac{\left[\frac{\partial \hat{q}}{\partial K_T} - \frac{\partial \hat{q}}{\partial K_N} \right] \frac{\hat{B}_{t+1} - (1-\gamma)\hat{B}_t}{\hat{P}_t}}{\mathbb{E}_t \left[\frac{\beta u'\left(\hat{C}_{t+1}\right)}{u'\left(\hat{C}_t\right)} \hat{R}_{N,t+1} \right]} \tag{23}$$

where all "hat" variables correspond to the efficient allocation.

The remainder of this section explores the quantitative properties of $\hat{q}$ under a standard parametrization, including a value of $\eta < 1$ which is the sufficient condition suggested from Proposition 1 for the second term in equation 22 to be positive. I also explore how this disagreement evolves over the business cycle and during commodity windfalls, which will suggest possible avenues for policy intervention.

3.4 Calibration

A period in the model corresponds to one quarter. I take most parameters from the literature, which are summarized in Table 1. The main purpose of this exercise is not to match any particular features of the data, but rather to illustrate how the theoretical results from the two-period model continue to hold quantitatively in the richer infinite-horizon environment with standard parameter values.

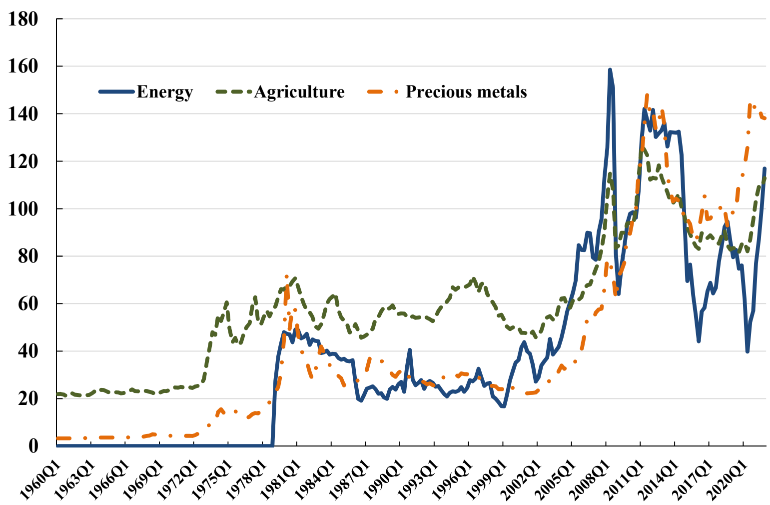
The risk-free interest rate is $r^* = 0.01$, the CRRA parameter is $\sigma = 2$, and the discount factor is $\beta = 0.98$, which are standard values in business cycle and sovereign default studies. The elasticity of substitution between traded and non-traded goods is $\eta = 0.83$ and the share of non-traded is ω; both of which I take from Bianchi (2011). The capital shares are $\alpha_N = \alpha_T = 0.33$, and the parameters governing the stochastic process for productivity are $\rho = 0.94$ and $\sigma_z = 0.027$, which are all standard values. I take the debt duration parameter $\gamma = 0.05$ and the coupon rate $\kappa = 0.03$ from Chatterjee and Eyigungor (2012).

Table 1: Infinite-horizon calibration

Parameter	Value	Parameter	Value
σ	2	β	0.98
r^*	0.01	ϕ	2.5
η	0.83	ω	0.6
α_N	0.33	α_T	0.33
d_0	-0.21	d_1	0.42
ρ	0.94	σ_z	0.027
$y_{C,L}$	0.11	$y_{C,H}$	0.25
$Pr\,(\text{windfall})$	0.05	windfall duration	16 quarters
γ	0.05	κ	0.03

For commodity windfalls I consider data from the World Bank Commodity Price Data. Figure 1 shows price indices for three main commodity categories: energy, agriculture, and precious metals.

Figure 1: Primary commodities prices, quarterly indices



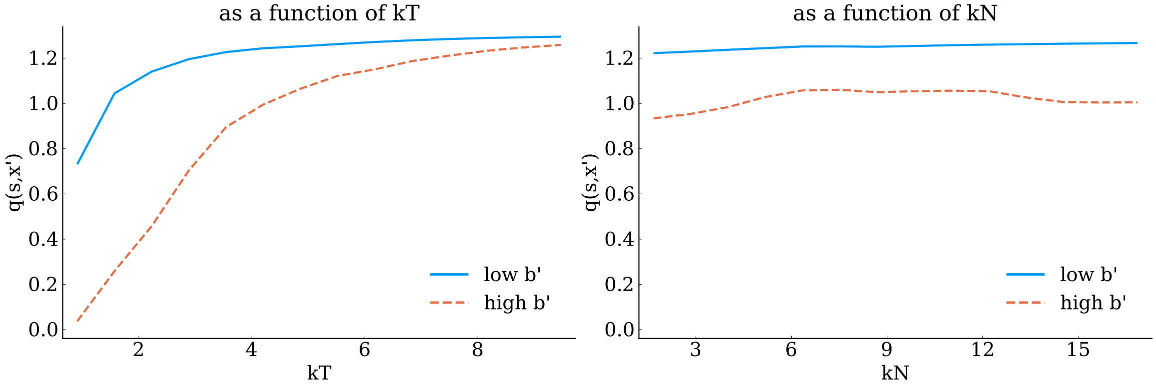
Indices are the quarterly average from monthly data. Energy includes coal, crude oil, natural gas, and liquified natural gas. Agriculture includes beverages, food, and raw materials. Precious metals includes gold, platinum, and silver. The data for energy start in January, 1979.

I define the start of a commodity windfall as a quarter in which the index increase relative to the previous 10 quarters is above the 95th percentile of such increases. Thus, I set the probability of a commodity windfall to $Pr(\text{windfall}) = 0.05$. Similarly, I define the end of a windfall as the quarter when the price index drops by at least half of the increase when the windfall started. This gives an average duration of 16 quarters.

3.5 Over-investment in non-traded goods

The quantitative version of the model allows for both stocks of capital to evolve endogenously. Each type of capital has two effects on the price of the government debt. The first has to do with the level of capital, which affects default incentives through the level of available resources in the future and its implications for consumption possibilities in default and repayment. The second, is the effect that the sectoral portfolio of capital has on default incentives. Figure 2 shows the price of the planner's debt as a function of both types of capital.

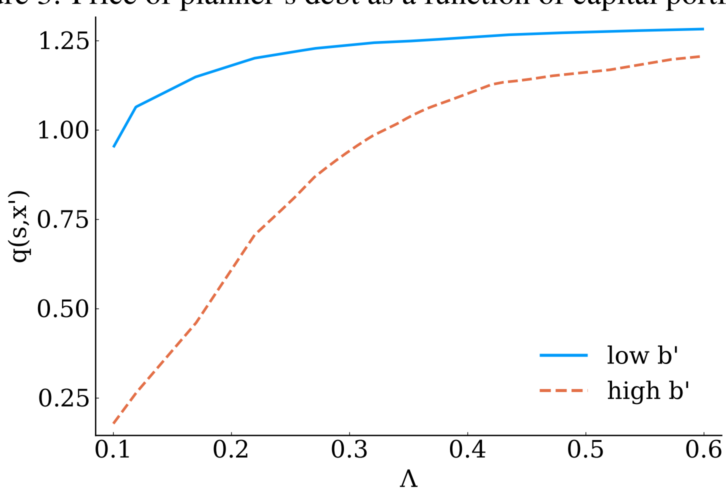
Figure 2: Price of the planner's debt



In both panels the price schedule is evaluated at the mean productivity shock and low commodity endowment. The left panel is evaluated at the steady state level of non-traded capital and the right panel is evaluated at the steady state level of traded capital.

These figures suggest that, under the current calibration, $\hat{q}$ is more sensitive to the level of capital for the traded good K_T. This suggests that the second term in equation 22 is positive for positive levels of debt issuance. That is, when the planner tries to issue additional debt, it faces more favorable borrowing terms if it switches capital from the traded to the non-traded sector. To make this point clearer and more comparable to Proposition 1, Figure 3 plots the price of the planner's debt as a function of the share of capital in the traded sector Λ for a fixed level of total capital $\bar{K} = K_T + K_N$. That is, it plots $\hat{q}(s, \Lambda K_T, (1-\Lambda) K_N, B)$ over different values of Λ.

Figure 3: Price of planner's debt as a function of capital portfolio



The price schedule is evaluated at the mean productivity shock and low commodity endowment. The aggregate level of capital $\bar{K}$ is the sum of steady-state levels for K_T and K_N.

The above Figures suggest that the second term in equation 22 is positive. Like in the two period model, borrowing terms improve if, for a given level of total capital in the economy, a bigger share of it is allocated to the traded sector. Then, from the planner's Euler equations, we have that

$$\mathbb{E}_t\left[\frac{\beta u'\left(\hat{C}_{t+1}\right)}{u'\left(\hat{C}_t\right)}\left(\hat{R}_{T,t+1}-\hat{R}_{N,t+1}\right)\right]\leq 0$$

under the efficient allocation. This implies that, as in the two-period model, the planner chooses to forego additional returns in the non-traded sector in order to be able to face more favorable borrowing terms. The households fail to internalize this and, in general, invests more capital in the non-traded sector than what would be efficient.

Table 2 compares business cycle moments between the planner and the decentralized economy. These are the average moments of 300 economies over 50 periods in their ergodic state. I only consider economies that had not experienced a default event for at least 25 periods before $t=1$, as is standard in the literature.

Table 2: Business cycle moments

	Planner	Decentralized
$r-r^*$	7.1%	12.3%
$Pr\,(\text{default})$	1.5%	3.0%
$\frac{\sigma_c}{\sigma_{GDP}}$	1.2	1.33
$\frac{\sigma_{inv}}{\sigma_{GDP}}$	3.8	4.1
$Cor\left(\frac{ca}{gdp},gdp\right)$	-0.44	-0.45
$Cor\,(r-r^*,gdp)$	-0.61	-0.32

These are the average moments of 300 economies over 50 periods in their ergodic state (I simulate each economy for 1050 periods and drop the first 1000). I only consider economies that had not experienced a default event for at least 25 periods.

In general, the government in the decentralized economy faces higher interest rate spreads and defaults more frequently. This is a result of the disagreement in the Euler equations for capital illustrated above. Consumption and investment are also more volatile in the decentralized economy. In both cases the current account and the spreads are counter-cyclical, as is standard in the literature. However, spreads are more counter-cyclical for the planner, indicating that the planner is more able to take advantage of low spreads during "good times" (i.e. in states where the default probability is low). This is true, in general, when income in the near future is expected to be high such as high realizations of productivity or during commodity windfalls.

3.6 The Dutch disease and optimal taxes

During persistent commodity windfalls (i.e. high y_C which is expected to remain high) there are two forces that are worth considering. The first is the classic Dutch disease effect in which high future traded income crowds capital out of the traded sector into the non-traded sector. This is because the real exchange rate appreciates with high y_C, which in turn increases the return to capital in the non-traded sector through a higher relative price of non-traded goods. The second is that a relatively impatient government (or planner) would like to front-load some consumption from the future high commodity income, which increases the incentives to issue more debt. In this environment, the Dutch disease amplifies the disagreement between the government and the households regarding the allocation of capital, hampering the government's ability to optimally increase borrowing.

To illustrate this, I compute time series of the optimal tax from equation 23 and compare its value during periods with low commodity income $y_C = y_{C,L}$ and during commodity windfalls $y_C = y_{C,H}$. Table 3 reports the optimal tax and default probabilities for the planner and the decentralized economy, conditional on each level of commodity income.

Table 3: Optimal taxes with low and high y_C

	$y_C = y_{C,L}$	$y_C = y_{C,H}$
average τ^*	2.4%	3.0%
Pr (default decentralized)	3.5%	1.5%
Pr (default planner)	2.6%	0.7%

I simulate 11,000 quarters and drop the first 1000. The left column averages each variable conditional on $y_C = y_{C,L}$ and the right column conditons on $y_C = y_{C,H}$.

As expected, the optimal tax is, on average, higher during commodity windfalls. This shows that, quantitatively, the intuition that the Dutch disease amplifies the inefficiency holds. Default probabilities are lower with high commodity income in both the decentralized economy and for the planner, however they are lower in general under the efficient allocation. Note that this result does not rest on any discrepancy in preferences between the government, the planner, or households. By assumption, the government is benevolent and the only difference between the government and the planner is their ability to pin down the investment portfolio in the economy.

4 Empirical analysis

This section makes two empirical points, which support the main theoretical findings. The first is that, in the long-run, resource-rich countries face more stringent borrowing terms. The second point is that central banks accumulate more international reserves during commodity windfalls in order to prevent over-appreciation of the real exchange rate. This is consistent with the quantitative result from Section 3 that the optimal tax τ^* is higher when commodity income is higher because a real depreciation lowers returns to capital in the non-traded in the same way as the optimal tax does.

4.1 Data description

Unless indicated otherwise, all data are yearly and taken from The World Bank (2021) and the International Monetary Fund (2021). I consider all countries with available data for the years 1979–2015.

I use two measures of default risk. The first is the interest rate spreads from JP Morgan's Emerging Markets Bonds Index (EMBI), which are widely used in the literature. These data are available for 37 countries starting no earlier than 1993.[12] As an alternative measure, I use the Institutional Investor Index (*III*) to construct measures of spreads for other countries for which sovereign bonds spread data are not available. The *III* is a measure of sovereign risk that was published biannually by the Institutional Investor magazine between 1979 and 2015. It measures country risk by aggregating into an index a collection of risk-related variables that are related to investing in a foreign country, including political risk, exchange rate risk, economic risk, sovereign risk and transfer risk. The *III* takes values between 0 and 100, where 100 indicates lowest risk and 0 the most risk. To assess how the *III* explains sovereign spreads, I estimate the following econometric model:

$$\ln\left(\text{spread}_{i,t}\right) = \gamma_0 + \gamma_1 \ln\left(III_{i,t}\right) + \kappa_i + \mu_t + \varepsilon_{i,t} \tag{24}$$

where κ_i are country fixed effects, μ_t are year fixed effects, $III_{i,t}$ is the average index for country i

[12] The 37 countries are: Argentina, Belize, Brazil, Bulgaria, Chile, China, Colombia, Dominican Republic, Ecuador, Egypt, El Salvador, Gabon, Ghana, Hungary, Indonesia, Iraq, Jamaica, Kazakhstan, Republic of Korea, Lebanon, Malaysia, Mexico, Pakistan, Panama, Peru, Philippines, Poland, Russian Federation, Serbia, South Africa, Sri Lanka, Tunisia, Turkey, Ukraine, Uruguay, Venezuela, and Vietnam.

in year t, and $\varepsilon_{i,t}$ is the error term.[13] I then use equation (24) and III data to construct time-series of spreads for all countries.

I use data on total natural resource rents as a fraction of GDP. Natural resource rents are calculated as the difference between the price of a commodity and the average cost of producing it. These unit rents are then multiplied by the physical quantities that countries extract to determine the rents for each commodity. Total natural resource rents are the sum of oil rents, natural gas rents, coal rents, other mineral rents, and forest rents.

I use two measures of foreign debt: total external debt stocks and central government debt, both as a fraction of GDP. The former includes both private and public debt, while the latter includes only government debt but is available for a smaller set of countries. I use international reserves excluding gold as a fraction of GDP and a measure of the real exchange rate described below.

4.2 Default risk and natural resources

First, to show the long-run relation between being a commodity exporter and spreads, I estimate the following panel regression:

$$s_{i,t} = \beta_0 + \beta_1 \overline{NR}_i + \beta_2 100 * \frac{debt_{i,t}}{GDP_{i,t}} + \beta_3 100 * \frac{reserves_{i,t}}{GDP_{i,t}} + \mu_t + u_{i,t} \tag{25}$$

where subscripts i refer to countries and t to years, $s_{i,t}$ are interest rate spreads, $\overline{NR}_i$ is the average natural resource rents as a percentage of GDP for country i over the available time period, μ_t are year fixed effects, and $u_{i,t}$ is the error term. Table 4 summarizes the estimation results for different measures of spreads and government debt.

[13] The estimated coefficients are

$$\ln\left(\text{spread}_{i,t}\right) = \underset{(0.629)}{8.791} - \underset{(0.177)}{1.958} \ln\left(III_{i,t}\right)$$

where the numbers in parenthesis are clustered standard errors. The III is significant at the 0.01 level and the $R^2 = 0.64$.

Table 4: Commodity exporters and default risk

	(1)	(2)	(3)	(4)
	EMBI	EMBI	Constructed EMBI	Constructed EMBI
Av (NR rents / GDP)	0.128**	0.137	0.208**	0.926***
	(0.0605)	(0.125)	(0.0804)	(0.281)
Reserves / GDP	-0.124***	-0.132**	-0.360***	-0.0853***
	(0.0375)	(0.0481)	(0.0358)	(0.0285)
Total Debt / GDP	0.0678*		0.167***	
	(0.0332)		(0.0237)	
Gov Debt / GDP		0.0442**		0.122***
		(0.0198)		(0.0380)
Constant	4.330**	3.882***	4.438***	-5.040**
	(1.513)	(0.627)	(0.975)	(1.829)
Year FE	Yes	Yes	Yes	Yes
Observations	520	246	2,645	1,033
Number of countries	43	31	105	84
R-squared	0.267	0.307	0.216	0.292

Robust standard errors in parenthesis based on Driscoll and Kraay (1998).
*** p<0.01, ** p<0.05, * p<0.1

The first row shows that the estimates of β_1 are positive and statistically different from 0 (except for column (2), which has the least number of observations). The variable $\overline{NR}_i$ is a country-specific shifter and the positive sign of β_1 indicates that countries for which natural resource rents are relatively large face higher default risk for any given level of foreign debt and assets (since the regression controls for these variables). The estimates in column (4) indicate that a 1 percent higher share of rents from commodities on GDP implies that average government spreads are 92 basis points higher.

4.3 Real exchange rates and reserve accumulation

To explore the relation between accumulation of international reserves and commodity windfalls I estimate the following regression:

$$\ln\left(100 * \frac{reserves_{i,t}}{GDP_{i,t}}\right) = \chi_0 + \chi_1 \ln\left(100 * \frac{NR_{i,t}}{GDP_{i,t}}\right) + \kappa_i + \mu_t + v_{i,t} \tag{26}$$

where the dependent variable is the natural logarithm of international reserves as a percentage of GDP, $NR_{i,t}$ are rents from natural resources in country i in year t, κ_i are country fixed effects, μ_t are year fixed effects, and $v_{i,t}$ is the error term. Table 5 reports the estimated coefficients.

Table 5: Relation between reserves and commodity windfalls

	(1) Reserves
$\ln\left(100 * \frac{NR_{i,t}}{GDP_{i,t}}\right)$	0.117***
	(0.0333)
Constant	1.635***
	(0.0380)
Year FE	Yes
Country FE	Yes
Observations	5,044
Number of countries	160
R-squared	0.183

Robust standard errors in parenthesis based on Driscoll and Kraay (1998).
*** p<0.01, ** p<0.05, * p<0.1

There is a significant positive relation between rents from natural resources and international reserves, suggesting that in the presence of commodity windfalls central banks increase their reserve accumulation. Finally, to analyze the effect that natural resources and reserve acummulation have on real exchange rates, I estimate the following regression:

$$\ln(rer_{i,t}) = \rho \ln(rer_{i,t-1}) + \phi_1 \left(100 * \frac{NR_{i,t}}{GDP_{i,t}}\right) + \phi_2 \Delta_{t,t-1} \left(100 * \frac{reserves_{i,t}}{GDP_{i,t}}\right) + \kappa_i + \mu_t + \varepsilon_{i,t} \tag{27}$$

where κ_i are country fixed effects, μ_t are year fixed effects, $\Delta_{t,t-1}x_t$ indicates the change of variable x from $t-1$ to t, and $rer_{i,t}$ is the real exchange rate of country i in year t *vis-a-vis* the US dollar.[14] Table 6 reports the estimated coefficients from equation (27).

[14] I compute $rer_{i,t} = \frac{e_{i,t} P_{US,t}}{P_{i,t}}$ where $e_{i,t}$ is the nominal exchange rate (amount of currency from country i per US dollar), $P_{US,t}$ is the US GDP deflator in year t and $P_{i,t}$ is country i's GDP deflator.

Table 6: Effect of reserves and natural resources on the real exchange rate

	(1)
	Real Exchange Rate
$\ln\left(rer_{i,t-1}\right)$	0.909***
	(0.0272)
$\left(100 * \frac{NR_{i,t}}{GDP_{i,t}}\right)$	-0.00597**
	(0.00284)
$\Delta_{t,t-1}\left(100 * \frac{reserves_{i,t}}{GDP_{i,t}}\right)$	0.00203**
	(0.000833)
Constant	0.280***
	(0.0945)
Year FE	Yes
Country FE	Yes
Observations	3,980
Number of countries	158
R-squared	0.919

Robust standard errors in parenthesis based on Driscoll and Kraay (1998).
*** p<0.01, ** p<0.05, * p<0.1

The sign of $\phi_1 < 0$ indicates that the real exchange rate appreciates when rents from natural resources increase. The sign of $\phi_2 > 0$ indicates that the real exchange rate depreciates when reserve accumulation increases. Intuitively, the central bank increases the domestic demand for foreign currency in order to limit the effects of higher supply from higher commodity exports. This positive relation suggests that increasing the accumulation of international reserves tames the appreciation of the real exchange rate during commodity windfalls, which in turn decreases the distortion from the Dutch disease suggested by the model.

5 Conclusion

This paper presented an environment with production in traded and non-traded sectors in which, in the presence of default risk, atomistic households allocate higher than optimal amounts of capital to the non-traded sector. This misallocation of capital is a result of the private sector failing to internalize how its capital-allocation decisions affect ex-post default incentives and ex-ante borrowing terms. In addition, this misallocation is more severe in the presence of alternative sources of tradable income, such as large endowments of natural resources.

The efficient allocation can be decentralized as a competitive equilibrium with an appropriate tax to capital income in the non-traded sector. This tax is proportional to the desired borrowing level and to the sensitivity of borrowing terms to the investment portfolio between capital for traded and non-traded goods. In general, borrowing terms improve with relatively more capital in the traded sector, which justifies taxing returns to non-traded capital.

Sterilization policies such as accumulation of international reserves during commodity windfalls have effects that are consistent with the optimal tax: ex-post, they depreciate the real exchange rate and reduce the realized return to capital in non-traded sectors; ex-ante they reduce the incentives to overinvest in non-traded sectors, which reduces the capital misallocation highlighted by the model. The empirical evidence supports the two main implications from the model: (i) "resource-rich" economies face higher default risk, which is reflected in higher interest rate spreads, and (ii) the accumulation of international reserves increases during commodity windfalls, which depreciates the real exchange rate.

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A Euler equations for capital

A.1 Households

The problem of a representative household is:

$$\begin{aligned}
&\max_{\lambda_1} \int_{\underline{z}}^{z^*(x_1)} \beta u\left(c_1^D\right) dF(z) + \int_{z^*(x_1)}^{\bar{z}} \beta u(c_1)\, dF(z) \\
s.t.\quad & P^D c_1^D = \left[r_N^D (1-\lambda_1) + r_T^D \lambda_1\right] \bar{k} + y_{C,1} + \Pi_1^D \\
& P^P c_1 = \left[r_N^P (1-\lambda_1) + r_T^P \lambda_1\right] \bar{k} + y_{C,1} + \Pi_1^P - B_1 \\
& \Lambda_1 = \Gamma_H\left(B_1; y_{C,1}\right)
\end{aligned}$$

where the rental rates of capital are:

$$\begin{aligned}
r_N^D &= \left(\frac{\omega}{1-\omega} \frac{z_D(z)\left(\Lambda_1 \bar{K}\right)^{\alpha_T} + y_{C,1}}{z_D(z)\left((1-\Lambda_1)\bar{K}\right)^{\alpha_N}}\right)^{\frac{1}{\eta}} \alpha_N z_D(z)\left((1-\Lambda_1)\bar{K}\right)^{\alpha_N - 1} \\
r_N^P &= \left(\frac{\omega}{1-\omega} \frac{z\left(\Lambda_1 \bar{K}\right)^{\alpha_T} + y_{C,1} - B_1}{z\left((1-\Lambda_1)\bar{K}\right)^{\alpha_N}}\right)^{\frac{1}{\eta}} \alpha_N z\left((1-\Lambda_1)\bar{K}\right)^{\alpha_N - 1} \\
r_T^D &= \alpha_T z_D(z)\left[\Lambda_1 \bar{K}\right]^{\alpha_T - 1} \\
r_T^P &= \alpha_T z\left[\Lambda_1 \bar{K}\right]^{\alpha_T - 1}
\end{aligned}$$

and are taken as given by the household, as well as the prices:

$$\begin{aligned}
P^D &= \left[\omega\left(p_N^D\right)^{1-\eta} + (1-\omega)\right]^{\frac{1}{1-\eta}} \\
p_N^D &= \left(\frac{\omega}{1-\omega} \frac{z_D(z)\left(\Lambda_1 \bar{K}\right)^{\alpha_T} + y_{C,1}}{z_D(z)\left((1-\Lambda_1)\bar{K}\right)^{\alpha_N}}\right)^{\frac{1}{\eta}} \\
P^P &= \left[\omega\left(p_N^P\right)^{1-\eta} + (1-\omega)\right]^{\frac{1}{1-\eta}} \\
p_N^P &= \left(\frac{\omega}{1-\omega} \frac{z\left(\Lambda_1 \bar{K}\right)^{\alpha_T} + y_{C,1} - B_1}{z\left((1-\Lambda_1)\bar{K}\right)^{\alpha_N}}\right)^{\frac{1}{\eta}}
\end{aligned}$$

which all only depend on the aggregate state in period 1. The first-order condition of this problem is:

$$0 = \int_{\underline{z}}^{z^*(x_1)} u'\left(c_1^D\right) \frac{\partial c_1^D}{\partial \lambda_1} dF(z) + \int_{z^*(x_1)}^{\bar{z}} u'(c_1) \frac{\partial c_1}{\partial \lambda_1} dF(z)$$

where

$$\frac{\partial c_1^D}{\partial \lambda_1} = \frac{r_T^D - r_N^D}{P^D} \bar{K}$$
$$\frac{\partial c_1}{\partial \lambda_1} = \frac{r_T - r_N}{P} \bar{K}$$

do not depend on λ_1, only on $\Lambda_!$.

A.2 Social planner

The problem of the social planner is:

$$\max_{\Lambda_1, B_1} u(C_0) + \beta \int_{\underline{z}}^{z^*(x_1)} u\left(C^D\right) dF(z) + \beta \int_{z^*(x_1)}^{\bar{z}} u(C) dF(z)$$

using Leibniz integral rule we get that the first order condition for Λ_1 is:

$$u'(C_0) \frac{\partial C_0}{\partial \Lambda_1} + \beta \int_{\underline{z}}^{z^*(x_1)} u'\left(C_1^D\right) \frac{\partial C^D}{\partial \Lambda} dF(z) + \beta \int_{z^*(x_1)}^{\bar{z}} u'\left(C_1^P\right) \frac{\partial C^P}{\partial \Lambda} dF(z) = 0$$

It is easy to see from the firm's problems in the decentralized equilibrium that $\frac{\partial C^P}{\partial \Lambda} = \frac{r_T^P - r_N^P}{P^P} \bar{K}$ and $\frac{\partial C^D}{\partial \Lambda} = \frac{r_T^D - r_N^D}{P^D} \bar{K}$. The derivative of consumption in period 0 with respect to Λ_1 is:

$$\frac{\partial C_0}{\partial \Lambda_1} = \frac{\partial Y}{\partial c_T} \frac{\partial q}{\partial \Lambda_1} B_1$$

where $\frac{\partial q}{\partial \Lambda_1}$ is the derivative of the price of bonds with respect to Λ_1. It is easy to see from equation (8) that q is continuously differentiable over $[\underline{z}, \bar{z}]$ as long as z_D is continuously differentiable over $[\underline{z}, \bar{z}]$ and F is smooth.

B Proof of Proposition 2

PROPOSITION 2: (*Misallocation*) For any given level of debt issuance B_1, households in the decentralized equilibrium overinvest in the non-traded sector and underinvest in the traded one.

Proof: For $x = (\Lambda, B)$, recall that aggregate consumption in $t = 0$ is:

$$C_0(x) = Y\left(y_N\left(z_0, (1-\Lambda_0)\bar{K}\right), y_T\left(z_0, \Lambda_0\bar{K}\right) + y_C + q(x)B - B_0\right)$$

Define the continuation value of x as

$$V1(x) = \beta \int_{\underline{z}}^{z^*(x)} u\left(C^D(z,x)\right) dF(z) + \beta \int_{z^*(x)}^{\bar{z}} u\left(C^P(z,x)\right) dF(z)$$

and the value in $t = 0$ for the social planner of choosing x_1 as:

$$V0(x_1) = u(C_0(x_1)) + V1(x_1) \tag{28}$$

The problem of the social planner is to choose $\hat{x}_1$ to maximize 28.

Thus, we can express the F.O.C. of the planner's problem with respect to Λ is

$$0 = \frac{\partial V1(\hat{x}_1)}{\partial \Lambda} + u'\left(\hat{C}_0\right) \frac{\partial \hat{Y}}{\partial c_T} \frac{\partial \hat{q}}{\partial \Lambda_1} \hat{B}_1 \tag{29}$$

In equilibrium, all households choose the same capital allocation $\tilde{\lambda}_1^* = \tilde{\Lambda}_1$. Given some debt issuance B, the equilibrium allocation of capital $\tilde{\Lambda}_1(B)$is pinned down by the household's first-order condition, which can also be written in terms of the derivative of $V1$:

$$0 = \frac{\partial V1\left(\tilde{\Lambda}_1(B), B\right)}{\partial \Lambda} \tag{30}$$

We know from Proposition 1 that

$$\frac{\partial V1(\hat{x}_1)}{\partial \Lambda} \leq \frac{\partial V1\left(\tilde{\Lambda}_1\left(\hat{B}\right), \hat{B}\right)}{\partial \Lambda}$$

so since the objective is concave then its derivative is decreasing and we get that $\tilde{\Lambda}_1\left(\hat{B}\right) \leq \hat{\Lambda}_1$.